RUME XVI CONFERENCE SCHEDULE

Registration/Information Desk- 7:30 am - 5:30 pm (February 21-23, 2013)

THURSDAY

February 21, 2013

Working Groups
8:00-11:30

Working Group for Research on College Mathematics Instructor Professional Growth
Location: *Denver 5*
Organizers
Jessica Deshler, West Virginia University
Shandy Hauk, Wested & University of Northern Colorado

Working Group: Research on the Learning and Teaching of Combinatorics
Location: *Denver 6*
Organizers
Elise Lockwood, University of Wisconsin – Madison
Aviva Halani, Arizona State University

Working Group: Infinity and Limits in Undergraduate Mathematical Learning
Location: *Gold Coin*
Organizers
Rob Ely, University of Idaho
Timothy Boester, Wright State University

Working Group on Investigating Student Understanding of Cross-Cutting Concepts within Undergraduate Mathematics and Physics
Location: *Matchless*
Organizers
Megan Wawro, Virginia Tech
Warren Christensen, North Dakota State University

Working Group: Realistic Mathematics Education in Research in Undergraduate Mathematics Education
Location: *Colorado G*
Organizers
Jason Martin, University of Central Arkansas
Michael Oehrtman, University of Northern Colorado
Craig Swinyard, University of Portland

Working Group on Community College Mathematics Research
Location: *Colorado H*
Organizers
Irene M. Duranczyk, University of Minnesota
April Strom, Scottsdale Community College
John T. Smith, Pellissippi State Community College
Claire Wladis, Borough of Manhattan Community College at the City University of New York

Mark Yannotta, Clackamas Community College
Ann Sitomer, Portland Community College
Vilma Mesa, University of Michigan

OPENING SESSION
12:00 – 12:30 PM
Denver Ballroom 1-4

Session 1 – Preliminary Reports
12:40 – 1:10 PM

Wait a Minute…Is That Enough to Make a Difference?
Daniel Reinholz and Mary Pilgrim
Colorado G

Abstract: The "one-minute paper" (Stead, 2005) is a technique for facilitating communication between students and the teacher and promoting reflection. In this paper we focus on the types of questions students ask and how they may be related to success. We present preliminary results from an introductory university-level calculus course, indicating that the nature of questions asked by more successful and less successful students are different, suggesting that the types of reflections that students engage in may have a significant impact on the efficacy of such an intervention.

Rethinking Business Calculus in the Era of Spreadsheets
Mike May
Colorado H

Abstract: The author is Writing and electronic "book" to support the teaching of calculus to business students with the assumption that they will use a spreadsheet as their main computational engine. With the change in technology, it is appropriate to rethink the content of the course as a different technology makes different tasks accessible. This study looks at what the content of the course should be. It compares the official learning objects of the course, with de facto learning objectives obtained by analyzing final exams from 20 sections of the course, and with the results of a survey of the faculty of the business school, the client discipline. It is intended that this preliminary study establishes a baseline that can be used to evaluate the effectiveness of the new approach to the course.

Expert Performance on Routine and Novel Integral Application (Volume) Problems
Krista Toth and Vicki Sealey
Denver 5 & 6

Abstract: Past research has shown that students struggle when applying the definite integral concept, and these difficulties stem from incomplete understanding of the integral's underlying structure. This study aims to provide insight into the construction of effective mental structures for integrals by examining experts' solutions to volume problems. Seven mathematics faculty members from a large, public university solved three calculus-level volume problems (two routine, one novel) in videotaped interview sessions. Preliminary analysis shows that the experts have a rich understanding of definite integrals, and the few instances of errors seemed to be a

result of inattention as opposed to a deficit in understanding. Their problem-solving process was highly structured and detailed. The experts' visual representations varied from sparse and static to fully 3-dimensional and dynamic. We hope to use this and past student data to construct a framework for analyzing student understanding of integral volume problems.

How Pre-Service Teachers in Content Courses Revise Their Mathematical Communication

Nina White
Matchless

Abstract: Math content courses aim to develop mathematical reasoning and communication skills in future teachers. Instructors often assign problems requiring in-depth written explanations to develop these skills. However, when a student's conception is incorrect, does written feedback from the instructor create the cognitive dissonance necessary to effect realignment of the student's understanding? These conceptions may be mathematical ("what is a fraction?") or meta-mathematical ("what constitutes a justification?"). Assigning problem revisions theoretically creates space for cognitive dissonance by having students rethink their solutions. I investigate a revision assignment in a course for future teachers to understand the nature of students' revisions and the possible impetuses for these revisions. In particular, I find preliminary evidence that students' revisions demonstrate changes in their language, mathematics, and use of examples and representations. Further, students' adoption of new representations in their solutions are largely due to observing peers' presentations rather than to instructor feedback.

Opportunity to Learn from Mathematics Lectures

Emilie Wiesner, Tim Fukawa-Connelly, and Aaron Weinberg
Denver Ballroom 1-4

Abstract: Many mathematics students experience proof-based classes primarily through lectures, although there is little research describing what students actually learn from such classroom experiences. Here we outline a framework, drawing on the idea of the implied observer, to describe lecture content; and apply the framework to a portion of a lecture in an abstract algebra class. Student notes and interviews are used to investigate the implications of this description on students' opportunities to learn from proof-based lectures. Our preliminary findings detail the behaviors, codes, and competencies that an algebra lecture requires. We then compare those with how students behave in response to the same lecture with respect to sense-making and note-taking, and thereby how they approach opportunities to learn.

Assessment of Students' Understanding of Related Rates Problems

Costanza Piccolo and Warren Code
Gold Coin

Abstract: This study started with a thorough analysis of student work on problems involving related rates of change in a first-year differential calculus course at a large, research-focused university. In two sections of the course, students' written solutions to geometric related rates problems were coded and analyzed, and students' learning was tracked throughout the term. Three months after the end of term, "think-aloud" interviews were conducted with some of the

students who completed the course. The interviews and some of the written assessments were structured based on the classification of key steps in solving related rates proposed by Martin (2000). Our preliminary findings revealed a widespread, persistent use of algorithmic procedures to generate a solution, observed in both the treatment of the physical and geometric problem, and the approach to the differentiation, and raised the question of whether traditional exam questions are a true measure of students' understanding of related rates.

Session 2 – Contributed Reports
1:20 – 1:50 PM

Switcher and Persister Experiences in Calculus I

Jessica Ellis, Chris Rasmussen, and Kristin Duncan

Gold Coin

Abstract: Previous reports show that not only are too few students pursuing Science, Technology, Engineering, or Mathematics (STEM) fields, but also many who originally intend to pursue these fields leave after their experiences in introductory STEM courses. Based on data gathered in a national survey, we will present an analysis of 5381 STEM intending students enrolled in introductory Calculus in Fall 2010, 12.5% of whom switched out of a STEM trajectory after their experience in Calculus I. When asked why these students no longer intended to continue taking Calculus (an indicator of continuing their pursuit of a STEM major), 31.4% cited their negative experience in Calculus I as a contributing factor. We analyze student and their instructor survey responses on various aspects of their classroom experience in Calculus I to better understand what aspects of this experience contributed to their persistence.

Critiquing the Reasoning of Others: Devil's Advocate and Peer Interpretations as Instructional Interventions

Aviva Halani, Owen Davis, and Kyeong Hah Roh

Denver 5 & 6

Abstract: This study investigated the ways in which college mathematics teachers might encourage the development of student reasoning through critiquing activities. In particular, we focused on identifying situations in which the instructional interventions were implemented to encourage the critiquing of arguments and in which students explained another's reasoning. Data for the study come from two teaching experiments – one from the domain of combinatorics and the other from real analysis. Through open coding of the data, Devil's Advocate and Peer Interpretations emerged as effective interventions for the creation of sources of perturbation for the students and for assisting in the resolution of a state of disequilibrium. These two interventions differ in design and in the type of reasoning students evaluate, but they both provoke students to further develop their reasoning, and therefore their understanding. We discuss the implications of these interventions for both research and teaching practice.

Entity versus Process Conceptions of Error Bounds in Students' Reinvention of Limit Definitions

Robert Raish, Michael Oehrtman, Jason Martin, Brian Fisher, and Craig Swinyard

Denver Ballroom 1-4

Abstract: We report results from a guided reinvention of the definition of sequence convergence conducted in three second-semester calculus classes. This report contributes to the growing body of research on how students come to understand and reason with formal limit definitions, focusing on the emergence of students' understanding of the epsilon quantity, conceived in terms of error bounds. Using Sfard's framework of the condensation of processes to entities, we mapped the possible conceptual trajectories followed by the students in the study. In this report, we detail our map, these trajectories and students' reasoning about other aspects of the formal definition, and the influence of reasoning about approximations and error analyses in students' progression.

Perspectives that Some Mathematicians Bring to University Course Materials Intended for Prospective Elementary Teachers

Elham Kazemi and Yvonne Lai

Matchless (updated)

Abstract: Many elementary teachers receive their certification in undergraduate contexts, where they are taught mathematics content courses by mathematics faculty. However, there is a disconnect between courses typically taught in mathematics department, such as service courses for engineering or advanced mathematics courses, and mathematics content courses for teachers – often both in the instructors' experience with the material as well as in the way the courses are taught. In this paper, we report on four mathematicians' reviews of one set of materials for content courses for prospective elementary teachers. We report on perspectives these mathematicians brought to the materials regarding mathematics, mathematical knowledge for teaching, and teaching. We report on analysis of these perspectives for what may be visible and invisible about mathematical knowledge for teaching and the work of teaching.

Session 3 – Preliminary Reports
2:00 – 2:30 PM

Not All Informal Representations are Created Equal

Kristen Lew, Juan Pablo Mejia-Ramos, and Keith Weber

Denver Ballroom 1-4

Abstract: Some mathematics educators and mathematicians have suggested that students should base their proofs on informal reasoning (Garuti et al. 1998). However, the ways in which students implement informal representations are not well understood. In this study, we investigate informal representations made by undergraduates during proof construction. Their use of informal representations will be compared to mathematicians' use of informal representations as described in Alcock (2004) and Samkoff et al. (2012). Further, an analysis of different types of informal representations will investigate the necessity to treat these different representations more carefully in the future.

Fostering Students' Understanding of the Connection Between Function and Derivative: A Dynamic Geometry Approach

Dov Zazkis

Denver 5 & 6

Abstract: Students' difficulties with relating the graphs of functions to the graphs of their

derivatives have been well documented in the literature. Here I present a Geometer's Sketchpad based applet, which was used as part of a technologically enriched Calculus I course. Individual interviews with students conducted after this in-class activity show evidence of varied and powerful student problem solving strategies that emerged after participation in the activity.

Determining the Structure of Student Study Groups
Gillian Galle
Matchless

Abstract: Although students are expected to spend time outside the classroom furthering their understanding of the material, there has been little verification of what students actually do when they study. This project observed undergraduate students studying together outside of the classroom setting in order to determine what study groups formed and what structures described these groups. Elements of social network analysis were employed to identify the groups that students formed. Transcripts of the study sessions were coded and frequency counts were established for each type of student interaction in order to characterize the roles students assumed while studying. This paper discusses the process of identifying the study groups and sets the groundwork for sharing the student roles. One main finding of this work is that the presence of a student recognized as an authority or facilitator of the group impacts the type of conversations that occur in the group setting.

Assessing Pre-Service Teachers' Conceptual Understanding of Mathematics Using Praxis II Data
Revathi Narasimhan
Colorado H

Abstract: We summarize the preliminary results of a study of conceptual understanding of mathematics by pre-service secondary school math teachers. Our research involves the statistical analysis of data from an actual mathematics Praxis II licensure exam, which was administered nationwide. Through a quantitative, item by item analysis, using a classification of these test items by conceptual difficulty, we obtain insight into the conceptual issues that pre-service teachers have great difficulty with. Our preliminary results show a significant gap between computational and abstract mathematical processes. This in turn, affects the ability of pre-service teachers to be fluent in the domains of both subject and pedagogical content knowledge.

A Case Study on a Diverse College Algebra Classroom: Analyzing Pedagogical Strategies to Enhance Students' Mathematics Self-Efficacy
Michael Furuto and Derron Coles
Gold Coin

Abstract: Shifting demographics show America rapidly diversifying, yet research indicates that an alarming number of diverse students continue to struggle to meet learning outcomes of collegiate mathematics curriculum. Consequently, recruitment and retention of diverse students in STEM majors is a pervasive issue. Using a sociocultural perspective, this study examined the effect of two pedagogical strategies (traditional instruction and cooperative learning) in a diverse College Algebra course on enhancing students' mathematics self-efficacy. Particular attention

was paid to investigating the role student discourse and interaction play in facilitating learning, improving conceptual understanding, and empowering students to engage in future self-initiated communal learning. The goal is to develop an effective classroom model that cultivates advancement in content knowledge and enculturation into the STEM community, culminating in a higher retention rate of diverse students in STEM. Preliminary data analysis suggests that a hybrid model encompassing both traditional instruction and cooperative learning successfully enhances students' self-efficacy.

A Multidimensional Analysis of Instructional Practices
Melissa Mills
Colorado G

Abstract: This study is an investigation of the questions that are asked by four faculty members who were teaching advanced mathematics. Each question was analyzed along three dimensions: the expected response type of the question, the Bloom's Taxonomy level, and the context of the question within the mathematics content.

COFFEE BREAK
2:40 – 3:10 PM
Denver Ballroom Pre-Function Area

Session 4 – Contributed Reports
3:20 – 3:50 PM

Under the Radar: Foundational Meanings that Secondary Mathematics Teachers Need, Do Not Have, but Colleges Assume
Pat Thompson, Neil Hatfield, Cameron Byerly and Marilyn Carlson
Denver Ballroom 1-4

Abstract: High school mathematics teachers must have coherent systems of mathematical meanings to teach mathematical ideas well. One hundred five teachers were given a battery of items to discern meanings they held in with respect to quantities, variables, functions, and structure. This paper reports findings on a sample of items that, by themselves, should alert college mathematics professors that foundational understandings they assume students have in advanced mathematics courses likely are commonly missing.

Using Cognitive Science with Active Learning in a Large Lecture College Algebra Course
David Miller and Matthew Schraeder
Gold Coin

Abstract: At a research university near the east coast, researchers have restructured a College Algebra course by formatting the course into two large lectures a week, an active recitation size laboratory class once a week, and an extra day devoted to active group work called Supplemental Practice (SP). SP was added as an extra day of class where the SP leader has students work in groups on a worksheet of examples and problems, based off of worked-example research, that were covered in the previous week's class material. Two sections of the course were randomly chosen to be the experimental group and the other section was the control group. The experimental group was given the SP worksheets and the control group was given a question-

and-answer session. The experimental group significantly outperformed the control on a variety of components in the course, particularly when the number of SP days was analyzed.

On the Plus Side: A Cognitive Model of Summation Notation
Steve Strand and Sean Larsen
Denver 5 & 6

Abstract: This paper provides a framework for analyzing and explaining successes and failures when working with summation notation. Cognitively, the task of interpreting a given summation-notation expression differs significantly from the task of expressing a long-hand sum using summation notation. As such, we offer separate cognitive models that 1) outline the mental steps necessary to carry out each of these types of tasks and 2) provide a framework for explaining why certain types of errors are made.

Preservice Elementary Teachers' Understanding of Greatest Common Factor Story Problems
Kristen Noblet
Matchless

Abstract: Little is known about preservice elementary teachers' mathematical knowledge for teaching number theory concepts, like greatest common factor or GCF. As part of a larger case study investigating preservice elementary teachers' understanding of topics in number theory, both content knowledge and pedagogical content knowledge (Shulman, 1986), a theoretical model for how preservice elementary teachers understand GCF story problems was developed. An emergent perspective (Cobb & Yackel, 1996) was used to collect and analyze data in the form of field notes, student coursework, and responses to task-based one-on-one interviews. The model resulted from six participants' responses to three sets of interview tasks where participants discussed concrete, visual, and story problem representations of GCF. In addition to discussing the model and relevant empirical evidence, I suggest language with which to discuss GCF representations.

Session 5 – Preliminary Reports
4:00 – 4:30 PM

Crossing Community Boundaries: Collaboration Between Mathematicians and Mathematics Educators
Sarah Bleiler
Colorado H

Abstract: Effective mathematics teachers are able to make connections between mathematical content and pedagogy in their professional practice. One of the most readily prescribed approaches for facilitating teachers' ability to make such connections is through the development of collaborations between mathematicians and mathematics educators in venues related to teacher professional development. Most prior research related to collaborative endeavors between these two groups has focused on the products, rather than the process, of collaboration. In this preliminary research report, I present the results of an interpretative phenomenological case study that investigated the team-teaching experiences of a mathematician and a mathematics

educator within the context of an undergraduate mathematics teacher preparation program. I present extracts from interviews that highlight the instructors' perceptions related to crossing the boundaries of their professional communities of practice, and engage participants in discussion about relevant "boundary crossing" in their own institutional contexts.

Interplay Between Concept Image and Concept Definition: Definition of Continuity
Gaya Jayakody
Colorado G

Abstract: This study looks at the interplay between the concept image and concept definition when students are given a task that requires direct application of the definition of continuity of a function at a point. Data was collected from 37 first year university students. It was found that different students apply the definition to different levels, which varied from formal deductions (based on the application of the definition) to intuitive responses (based on rather loose and incomplete notions in their concept image).

An Examination of Proving Using a Problem Solving Framework
Milos Savic
Matchless

Abstract: A link between proving and problem solving has been well established in the literature (Furinghetti & Morselli, 2009; Weber, 2005). In this paper, I discuss similarities and differences between proving and problem solving by using the Multidimensional Problem-Solving Framework created by Carlson and Bloom (2005) on Livescribe pen data from a study of proving (Author, 2012). I focus on two participants' proving processes: Dr. G, a topologist, and L, a mathematics graduate student. Many similarities were revealed by using the Carlson and Bloom framework, but also some differences distinguish the proving process from the problem-solving process. In addition, there were noticeable differences between the proving of the mathematician and the graduate student. This study may influence a proving-process framework that can encompass both the problem-solving aspect of proving and the differences found.

Talking Mathematics: An Abstract Algebra Professor's Teaching Diaries
Sepideh Stewart, John Paul Cook, Ralf Schmidt, and Ameya Pitale
Denver 5 & 6

Abstract: The world of a mathematician, with all its creativity and precision is fascinating to most people. This study is an account of collaboration between mathematicians and mathematics educators. In order to examine a mathematician's daily activities, we have primarily employed Schoenfeld's goal-orientated decision making theory to identify his Resources, Orientations and Goals (ROGs) in teaching an abstract algebra class. Our preliminary results report on a healthy and positive atmosphere where all involved freely express their views on mathematics and pedagogy.

Scaling Up Reinvention: Developing a Framework for Instructor Roles in the Classroom
Jungeun Park, Jason Martin, and Michael Oehrtman
Gold Coin

Abstract: Studies have shown that students have difficulty with the concept of limit, especially when reasoning about formal limit definitions. We conducted a five-day teaching experiment (TE) in a second semester calculus classroom in which students were asked to reinvent a formal sequence convergence definition. Author 3 (2011) detailed how pairs of students reinvented sequence convergence definitions but did not attempt the same instructional heuristic in the classroom. Our analysis focused on the instructor prompts and the TE students' subsequent group discussion through their use of key words and visuals in revising their definition. An interview with the instructor was conducted to investigate his intention of using specific prompts and his thinking about the TE group's choice of words and visuals. In our preliminary analysis, we found that the roles of the instructor were extended beyond those roles previously reported as roles for facilitators with pairs of students.

An Analysis of First Semester Calculus Students' Use of Verbal and Written Language When Describing the Intermediate Value Theorem

Vicki Sealey and Jessica Deshler

Denver Ballroom 1-4

Abstract: This preliminary report describes the second stage of data collection and analysis in a larger study that examines students' written and verbal language when studying basic theorems in a first-semester calculus course. We examine students' difficulties with understanding and using mathematical language and notation in both formal written work and informal verbal descriptions. Not surprisingly, the students in our study rarely use formal mathematical language without being prompted to do so. One surprising result was that while many students do understand the mathematical notation in the theorems, and can illustrate this graphically when prompted, they still do not use this notation when providing their own written (or verbal) description of a theorem. Preliminary results suggest that our biggest obstacle as teachers is not in getting our students to understand the notation, but instead lies in convincing our students of the power that comes from this notation in describing a concept, thus encouraging our students to use this notation in their own written work.

Session 6 – Contributed Reports
4:40 – 5:10 PM

Students' Conceptions of Mathematics as a Discipline

George Kuster

Colorado H

Abstract: Researchers have found that students' beliefs about mathematics impact the way in which they learn and approach mathematics in general. The purpose of this study is to categorize college students' various conceptions concerning mathematics as a discipline. Results from this study were used to create a preliminary framework for categorizing student conceptions. The results of this study indicate that the conceptions are numerous and range greatly in complexity. The results also suggest the need for further study to qualify the various student conceptions and the roles they play in students' understanding of and approach to performing mathematics.

Using Disciplinary Practices to Organize Instruction of Mathematics Courses for

Prospective Teachers
Yvonne Lai
Matchless

Abstract: One challenge of teaching content courses for prospective teachers is organizing instruction in ways that represent the discipline with integrity while serving the needs of future teachers—for example, choosing math problems that provide a logical development of a topic while also addressing mathematical knowledge for teaching. This paper examines the work entailed in structuring in-class work in mathematics courses for teachers. It argues that practices of teaching that are mathematical—such as representing ideas, grounding reasoning in mathematical observations available to the class, using definitions, or using mathematical language—can be used to negotiate mathematical and pedagogical aims, and therefore can be used to organize instruction of mathematical knowledge for teaching while simultaneously developing a disciplinary understanding.

On the Emergence of Mathematical Objects: The Case of *e*^(*az*)
Ricardo Nemirovsky and Hortensia Soto-Johnson
Gold Coin

Abstract: In this report we propose an alternate account of mathematical reification as compared to Sfard's (1991) description, which is characterized as an "instantaneous quantum leap", a mental process, and a static structure. Our perspective is based on two in-service teachers' exploration of the function , using Geometer's Sketchpad. Using microethnographic analysis techniques we found that the long road to beginning to reify the function entailed interplay between body-generated motion and object self-motion, kinesthetic continuity between different sides of the "same" thing, cultural and emotional background of life with things-to-be, and categorical intuitions. Our results suggest that perceptuomotor activities involving technology may serve as an instrument in facilitating reification of abstract mathematical objects such as complex-valued functions.

Commonly Identified Students' Misconceptions About Vectors and Vector Operations
Aina Appova and Tetyana Berezovski
Denver 5 & 6

Abstract: In this report we present the commonly identified error patterns and students' misconceptions about vectors, vector operations, orthogonality, and linear combinations. Twenty three freshmen students participated in this study. The participants were non-mathematics majors pursuing liberal arts degrees. The main research question was: What misconceptions about vector algebra were still prevalent after the students completed a freshmen-level linear algebra course? We used qualitative data in the form of artifacts and students' work samples to identify, classify, and describe students' mathematical errors. Seventy four percent of students in this study were unable to correctly solve a task involving vectors and vector operations. Two types of errors were commonly identified across the sample: a lack of students' understanding about vector operations and projections, and a lack of understanding (or distinction) between vectors and scalars. Final results and conclusions include research suggestions and practitioner-based implications for teaching linear algebra in high school and college.

POSTER SESSION (Cash Bar)
5:30 – 6:30 PM
Denver Ballroom Pre-function Area

That's Nice …but is it worth sharing
Daniel Reinholz

Abstract: "Group-worthy" problems (cf. Featherstone et al., 2011) are nontrivial, often have multiple solution paths and require multiple competencies; in short, they provide opportunities for students to engage in meaningful groupwork. In contrast, many standard tasks degenerate into one student "teaching" the other, because the tasks do not have proper affordances to support collaborative learning. I present on "peer-worthy" problems, a set of problems that are useful for pairs of students to work on in settings other than full-blown collaborative groupwork. Peer-worthy problems should satisfy a number of the following criteria; they: (1) are nontrivial, (2) have multiple solution paths, (3) require students to generate examples, and (4) involve explanation. In this poster I contrast student interactions around two problems - one peer-worthy, the other a standard task.

The Flipped Classroom Model for College Algebra: Effects on Student Achievement
Jerry Overmyer

The past few years have seen a substantial rise in the use and interest in a teaching and learning paradigm most commonly known as the flipped classroom. It is called the flipped class model because the whole classroom/homework paradigm is "flipped". In its simplest terms, what used to be classwork (the lecture) is done at home via teacher-created videos and what used to be homework (assigned problems) is now done in class. This quantitative research compares 5 sections (n = 144) of college algebra using the flipped classroom methods with 6 sections (n = 181) of traditional college algebra and its effect on student achievement as measured through a common final exam. The data will be analyzed using ANOVA with interactions of the intervention measured with gender and ACT mathematics scores.

An Annotation Tool Designed to Interface with Webwork:
Interpreting Students' Written Work
Nicole Engelke, Gulden Karakok, and Aaron Wangberg

Abstract: We present how we are using tablets with an open-source online homework system to collect students' written work to calculus problems. The new whiteboard feature captures all student written work in real time. An annotation tool has also been incorporated into the system. Through this tool, we are examining how students solve chain rule problems and what actions they take to correct their mistakes. At this poster, we will allow users to try out the annotation tool and provide results of how we have used it to date.

Reasoning Abilities that Support Students in Developing Meaningful Formulas to Relate Quantities in an Applied Problem Context
Bethany Fowler, Kristin Frank, Hyunkyoung Yoon and Marilyn Carlson

Abstract: This poster illustrates and describes student thinking when responding to applied problems to relate two quantities that cannot be directly related by a single formula. Students who understood the meaning of the directive to define one quantity in terms of another, and who also conceptualized variables as representing varying values that a quantity can assume were

successful in constructing a meaningful formula to relate the values of two quantities that cannot be directly related by a single formula. Students who failed to construct meaningful formulas during their solution process either held the view that a variable is an unknown value to be solved for or could not meaningfully interpret the directive of the problem statement.

Coaching the Coaches: Supporting University Supervisors in the Supervision of Elementary Mathematics Instruction

Stefanie Livers

Abstract: This program evaluation study examines the impact of providing professional development on coaching strategies and mathematics pedagogy to university supervisors' on their supervision practice and teacher candidates' instructional practice and beliefs about mathematics. The mixed-methods study was designed to answer the following two questions: What are the effects of training university supervisors in mathematics pedagogy and coaching strategies on their supervision practices of elementary teacher candidates? What are the effects of training university supervisors in mathematics education and coaching strategies on elementary teacher candidates' instructional practices and their beliefs about mathematics?
The qualitative data consisted of supervisors' background experience, observations, and interviews. Quantitative data included teacher candidates' performance on the Reformed Observation Teaching Protocol (RTOP) and belief scores from the Mathematics Beliefs Instrument (MBI). Analysis of the data revealed that university supervisors' support changed as a result of the professional development, thus changing the beliefs of teacher candidates.

PLENARY SESSION & DINNER
6:30 – 8:30 PM
Denver Ballroom 1-4

The Role and Use of Examples in Learning to Prove

Dr. Eric Knuth
University of Wisconsin, Madison

Abstract: Proof is central to mathematical practice, yet a perennial concern in mathematics education is that students of all ages struggle to understand the nature of evidence and justification in mathematics. Mathematics education scholars have suggested that overreliance on examples to justify the truth of statements is an underlying reason for students' difficulties learning to prove. As such, example-based reasoning has typically been viewed as a stumbling block to overcome. My colleagues and I, however, view example-based reasoning as an important object of study and posit that examples play both a foundational and essential role in the development, exploration, and understanding of conjectures, as well as in subsequent attempts to develop proofs of those conjectures. In this talk, I will discuss research related to the role and use of examples in learning to prove as well as implications for teaching mathematics at both the secondary and tertiary levels.

FRIDAY

February 22, 2013

Session 7 – Preliminary Reports
8:30 – 9:00 AM

Comparing a "Flipped" Instructional Model in an undergraduate Calculus III Course

Nicholas Wasserman, Scott Norris and Thomas Carr
Colorado C

Abstract: In this small comparative study, we explore the impact of "flipping" the instructional delivery of content in an undergraduate Calculus III course. Two instructors collaborated to determine daily content and lecture notes; one instructor altered the instructional delivery of the content (not the content itself), utilizing videos to communicate procedural course content to students out-of-class, with time in-class spent on conceptual activities and homework problems. With similar numbers (n=41 and n=40) and types of students in each class, student performance on tests for both classes will be compared to determine any significant differences in achievement related to "flipping" the instructional delivery of content.

Development and Analysis of a Basic Proof Skills Test
Sandra Merchant and Andrew Rechnitzer
Denver 5 & 6

Abstract: We have developed a short (16 question) basic skills test for use in our institution's transition-to-proof course that assesses basic skills required to succeed in such a course. Using this test in our core introductory proof course, we have found that students are generally deficient in a number of skills assumed by instructors. In addition, using this test as a pre/post-test we have found that in this course students are learning some concepts well, but that learning gains on other concepts are much below desired levels. Finally, administration of the test to students in a higher level course has allowed us to assess retention of these skills. At this preliminary stage these skills appear to be retained into higher-level proof courses, but more data collection is needed, as well as a more extensive instrument to assess proof skills, rather than simply basic logic and comprehension.

A Microgenetic Study of One Students' Sense Making About the Temporal Order of Delta and Epsilon
Aditya Adiredja
Gold Coin

Abstract: The formal definition of a limit, or the epsilon delta definition is a critical topic in calculus for mathematics majors' development and the first chance for students to engage with formal mathematics. This report is a microgenetic study of one student understanding of the formal definition focusing on a particularly important relationship between epsilon and delta. diSessa's Knowledge in Pieces and Knowledge Analysis provide frameworks to explore in detail the structure of students' prior knowledge and their role in learning the topic. The study documents the progression of the student's claims about the dependence between delta and epsilon and explores relevant knowledge resources.

An Investigation of Pre-Service Secondary Mathematics Teachers' Development and Participation in Argumentation
Lisa Rice
Matchless

Abstract: This study investigates how two professors and pre-service secondary mathematics teachers engage in argumentation and proof in two courses. One course under investigation is a geometry course; the second is a methods of teaching mathematics course. The research also studies the how professors and pre-service teachers construct arguments and proofs. Examining the classroom discourse to understand how it may impact argumentation practices is another aspect of the research. Case study and grounded theory approaches are used to guide the data collection and analysis. Some data collected include interviews with the two professors and pre-service teachers and observations of the two courses and the pre-service teachers' classrooms during their student teaching. Data analysis so far indicates the geometry professor engages students in argumentation and proof in multiple ways.

Emergent Modeling and Riemann Sum

Kritika Chhetri and Jason Martin
Denver Ballroom 1-4

Abstract: This research focuses on mental challenges that students face and how they resolve these challenges while transiting from intuitive reasoning to constructing a more formal mathematical structure of Riemann sum while modeling "real life" contexts. A pair of Calculus I students who had just received instruction on definite integral defined using Riemann sums and illustrated as area under the curve participated in multiple interview sessions. They were given contextual problems related to Riemann sums but were not informed of this relationship. Our intent was to observe students' transitioning from model of to model for reasoning while modeling these problem situations. Results indicate that students conceived of five major conceptions during their first task and their reasoning from the first task that became a model for reasoning about their next task. In this paper we detail those conceptions and their reasoning that became model for reasoning on the second task.

Students' Way of Thinking About Derivative and its Correlation to Their Ways of Solving Applied Problems

Shahram Firouzian
Colorado D

Abstract: Previous researchers have examined students' understanding of derivative and their difficulties in solving applied problems and/or their difficulties in applying the basic knowledge of derivative in different contexts. There has not been much research approaching students' ways of thinking about derivative through the lens of applied questions. In this research, first I categorized the students' way of thinking about the basic concept of derivative by running a survey of questions addressing the different ways of thinking about derivative based on the existing research works. While analyzing these surveys, I used grounded theory and added more ways of thinking about derivative. I specially noticed very incomplete ways of thinking about derivative as described below. Since my goal was looking at the students' ways of thinking about derivative through the lenses of applied questions, I also piloted my applied questions survey with 51 multivariable calculus students. I noticed a lot of students struggling with defining variables (the initial translation as described below) and if they could define the variable, a lot of them struggled on applying their ways of thinking about derivative into solving the applied problem. These difficulties are great venues to study their ways of thinking about derivative

using their struggle in the applied questions. This is a summary of my initial works on this ongoing research, the goal of which is to shed new insights into students' solving of applied problems.

Session 8 – Contributed Reports
9:10 – 9:40 AM

Preparing Students for Calculus

April Brown Judd and Terry Crites
Matchless

Abstract: This quantitative study compared the implementation of a problem-based curriculum in precalculus and a modular-style implementation of traditional curriculum in precalculus to the historical instructional methods at a western Tier 2 public university. The goal of the study was to determine if either alternative approach improved student performance in precalculus, improved student efficacy around learning mathematics and better prepared students for success in a calculus sequence. The study used quantitative data collection and analysis. Results indicate students who experienced the problem-based curriculum should be better prepared to learn calculus but mixed results in terms of retention and success in calculus.

Partial Unpacking and Indirect Proofs: A Study of Students' Productive Use of the Symbolic Proof Scheme

Stacy Brown
Denver 5 & 6

Abstract: This paper examines mathematics majors' evaluations of indirect proofs and of the compound statements used in such forms of proofs. Responses to survey items with a cohort of 23 students and six 1-hour clinical interviews, indicate that the students who could successfully evaluate indirect arguments and who could successfully recognize logically equivalent statements, tended to use partially unpacked (Selden & Selden, 1995) versions of the statement and the proofs and, in so doing, demonstrated a productive use of the symbolic proof scheme. Whereas, both successful and unsuccessful students tended to use proof frameworks (Selden & Selden, 1995). Moreover, successful students' approaches are suggestive of activities, which are rarely found in introductory proof texts, yet may benefit novice proof writers.

Utilizing Types of Mathematical Activities to Facilitate Characterizing Student Understanding of Span and Linear Independence

Megan Wawro and David Plaxco
Denver Ballroom 1-4

Abstract: The purpose of this study is to investigate students' concept images of span and linear (in)dependence and to utilize the mathematical activities of defining, example generating, problem solving, proving, and relating to provide insight into these concept images. The data under consideration are portions of individual interviews with linear algebra students. Grounded analysis revealed a wide range of student conceptions about the span and/or linear (in)dependence. The authors organized these conceptions into four categories: travel, geometric, vector algebraic, and matrix algebraic. To further illuminate participants' conceptions of span

and linear (in)dependence, the authors developed a framework to classify the participants' engagement into five types of mathematical activity: defining, proving, relating, example generation, and problem solving. This framework could prove useful as a means of providing finer-grained analyses of students' conceptions and the potential value and/or limitations of such conceptions in certain contexts.

Coherence from Calculus to Differential Equations
Jennifer Czocher, Jenna Tague, and Greg Baker
Gold Coin

Abstract: Despite recent research efforts to make calculus more coherent with other fields, instructors still express dissatisfaction in the mathematical preparation of their students. Even further, we suggest that there are coherence issues within the field of mathematics. In this paper, we expose and examine an epistemological mismatch between how calculus is expected to be known in calculus and how calculus is expected to be used in differential equations.

COFFE BREAK
9:50 – 10:20 AM
Denver Ballroom Pre-Function Area

Session 9 – Contributed Reports
10:30 – 11:00 AM

Odd Dialogues on Odd and Even Functions
Dov Zazkis
Matchless

Abstract: A group of prospective mathematics teachers was asked to imagine a conversation with a student centered on a particular proof regarding the derivative of even functions and produce a script of this imagined dialogue. These scripts provided insights into the script-writers' mathematical knowledge, as well as insights into what they perceive as potential difficulties for their students, and by extension difficulties they may have had themselves when learning the concepts. The paper focuses on the script-writers' understandings of derivative and of even/odd functions.

Secondary Teachers' Development of Quantitative Reasoning
David Glassmeyer, Michael Oehrtman, and Jodie Novak
Gold Coin

Abstract: This study was designed to document the development of teachers' ways of thinking about quantitative reasoning, one of the standards for mathematical practice in the Common Core State Standards. Using a models and modeling perspective, the authors designed a model-eliciting activity (MEA) that was implemented in a graduate mathematics education course focusing on quantitative reasoning. Teachers were asked to create a quantitative reasoning task for their students, which they subsequently revised three times in the course after receiving instructor, peer, and student feedback. The MEA documented the development of the teachers' models of quantitative reasoning, and the findings of this study detail one group of three teachers' development over the course. Findings include an overall model of teachers'

development that is both generalizable and sharable for other researchers and teacher educators.

Students Reconciling Notions of One-to-One Across Two Contexts
Michelle Zandieh, Jessica Ellis, and Chris Rasmussen
Denver Ballroom 1-4

Abstract: This research is part of a larger study. In previous work we created a framework for analyzing student understanding that incorporates five clusters of metaphorical expressions as well as properties and computations that students spoke about when discussing function or linear transformation. In this paper we apply this framework to the setting of students reconciling their understandings of one-to-one in the context of precalculus-type functions with their understandings of one-to-one in the context of linear algebra. Ideally we would like students to be able to recognize a similar structure for one-to-one in each context, and thereby to strengthen their overall understanding of the notion of one-to-one. This proposal provides four vignettes that we found illustrative of the way students reasoned about one-to-one within and across the two contexts. More broadly we find the case of one-to-one as prototypical of the struggles students have in seeing similarities across contexts.

On Mathematics Majors' Success and Failure at Transforming Informal Arguments into Formal Proofs
Bo Zhen, Juan Pablo Meija-Ramos, and Keith Weber
Denver 5 & 6

Abstract: In this paper, we examine 26 instances in which mathematics majors attempted to write a proof based on an informal explanation. In each of these instances, we represent students' informal explanations using Toulmin's (1958) scheme, we use Stylianides' (2007) conception of proof to identify what one would need to accomplish to transform the informal explanation into a proof. We then compare this to the actions that the participant took in attempting to make this transformation. The results of our study are categories of actions that led students to successfully construct valid proofs and actions that may have hindered proof construction.

Session 10 – Contributed Reports
11:10 – 11:40 AM

Mathematicians' Example-Related Activity When Proving Conjectures
Elise Lockwood, Amy B. Ellis and Eric Knuth
Denver 5 & 6

Abstract: Examples play a critical role in mathematical practice, particularly in the exploration of conjectures and in the subsequent development of proofs. Although proof has been an object of extensive study, the role that examples play in the process of exploring and proving conjectures has not received the same attention. In this paper, results are presented from interviews conducted with six mathematicians. In these interviews, the mathematicians explored and attempted to prove several mathematical conjectures and also reflected on their use of examples in their own mathematical practice. Their responses served to refine a framework for example-related activity and shed light on the ways that examples arise in mathematicians' work. Illustrative excerpts from the interviews are shared, and five themes that emerged from the

interviews are presented. Educational implications of the results are also discussed.

Transfer of Critical Thinking Disposition from Mathematics to Statistics
Hyung Kim and Tim Fukawa-Connelly
Gold Coin

Abstract: In this study we draw on the constructs of eagerness, flexibility and willingness to characterize the necessary disposition for critical thinking that is required in learning statistics in addition to specific content knowledge (Enis, 1989). We investigated the challenges that students who are highly successful in mathematics might have in doing statistics and found that while a student might have an inquisitive disposition and good proficiency with the foundational mathematical concepts such as functions and function transformations, that same student might struggle in statistics. Even concepts that are seemingly related to their mathematical counterparts such as what is a variable when considering a population and sample may cause problems as the question is distinct enough from the mathematical sense. We suggest that such students may experience greater than usual affective problems in a statistics class and may, therefore, give up easier and earlier than students who were less successful mathematically.

The Emergence of Algebraic Structure: Students Come to Understand Zero-Divisors
John Paul Cook
Colorado D

Abstract: Little is known about how students learn the basic ideas of ring theory. While the literature addressing student learning of group theory is certainly relevant, the concept of zero-divisor in particular is one for which group theory has no analog. In order to better understand how students come to understand zero-divisors, this talk will present results from a study that investigated how students can capitalize on their intuitive notions of solving equations to reinvent the definitions of ring, integral domain, and field. In particular, the emergence and progressive formalization of the concept of zero-divisor at various stages of the reinvention process will be detailed and discussed.

Performance and Persistence Among Undergraduate Mathematics Majors
Joe Champion and Ann Wheeler
Matchless

Abstract: There is little mixed methods research into the patterns of course taking, performance, and persistence among mathematics majors, in general, and among secondary mathematics majors, in particular. Drawing from a sample of 42,825 mathematics enrollment records at two universities over a six-year period, this study presents quantitative summaries of mathematics majors' performance and persistence in undergraduate mathematics courses alongside qualitative themes from interviews of nine secondary mathematics majors at one of the universities. Implications include potential strategies for mathematics programs and faculty to support the success of mathematics majors in undergraduate mathematics coursework, with special emphasis on prospective secondary mathematics teachers.

LUNCH
11:50 – 12:50 AM

Denver Ballroom 1-4

Session 11 – Theoretical Reports
1:00 – 1:30 PM

PAR for the Course: Developing Mathematical Authority

Daniel Reinholz
Matchless

Abstract: Perceived mathematical authority plays an important role in how students engage in mathematical interactions, and ultimately how they learn mathematics. This paper elaborates the concept of mathematical authority (Engle, 2011) by introducing two concepts: scope and relationality. This elaborated view is applied to a number of peer-interactions in a specialized peer-assessment context. In this context, self-perceived authority influenced the way feedback was framed (as either questions or assertions).

Adapting Model Analysis for the Study of Proof Schemes

David Miller and Todd Cadwalladerolsker
Gold Coin

Abstract: This theoretical paper describes model analysis and our adaptation of this method to the study of proof schemes in a transition to proof course. Model analysis accounts for the fact that students may hold more than one idea or conception at a time, and may use different ideas and concepts in response to different situations. Model analysis is uniquely suited to study students' proof schemes, as students often hold multiple, sometimes conflicting proof schemes, which they may use at different times. Model analysis treats each student's complete set of responses as a data point, rather than treating each individual response as a separate data point. Thus, model analysis can capture information on the self-consistency of a student's responses. Data was collected in a Transition to Proof course and analyzed using both traditional descriptive statistics and model analysis. We find that model analysis offers significant insights not offered by traditional analysis.

Implications of Realistic Mathematics Education for Analyzing Student Learning

Estrella Johnson
Denver 5 & 6

Abstract: The primary goal of this work is to articulate a theoretical foundation based on Realistic Mathematics Education (RME) that can support the analysis of student learning, both individual and collective, by documenting changes in local activity. To do so, I will build on previous work on the analytic implications of the Emergent Perspective, such as Rasmussen and Stephan's (2008) analytic approach to documenting the establishment of classroom mathematical practices. The Emergent Perspective is broadly consistent with RME, but the existing analytic methods related to the Emergent Perspective fail to draw on the theoretical constructs provided by RME. For instance, current analytic methods fail to draw on the RME Emergent Models heuristic to inform the analysis of the development of mathematical practices related to models of/for student mathematical activity. Here I will be explicitly considering the roles that RME constructs could play in analytic processes consistent with the Emergent Perspective.

Developing Hypothetical Learning Trajectories for Teachers' Developing Knowledge of the Test Statistic in Hypothesis Testing

Jason Mark Dolor
Colorado C

Abstract: In the past decade, educators and statisticians have made new suggestions for teaching undergraduate statistics, in light of these new recommendations it is important to (re)evaluate how individuals come to understand statistical concepts and how such research should impact curricular efforts. One concept that plays a major role in introductory statistics is hypothesis testing and the computation of the test statistic to draw conclusions in a hypothesis test. This proposal presents a theoretical approach through the development of a hypothetical learning trajectory of hypothesis testing by utilizing sampling distributions as the building block of coming to understand statistical inference. In addition, this proposal presents a way this hypothetical learning trajectory may support the development of research-based curricula that foster an understanding of the test statistic and its role in hypothesis testing.

Session 12 – Preliminary Reports
1:40 – 2:10 PM

Computational Thinking in Linear Algebra

Spencer Bagley and Jeff Rabin
Denver 5 & 6

Abstract: In this work, we examine students' ways of thinking when presented with a novel linear algebra problem. We have hypothesized that in order to succeed in linear algebra, students must employ and coordinate three modes of thinking, which we call computational, abstract, and geometric. This study examines the solution strategies that undergraduate honors linear algebra students employ to solve the problem, the variety of productive and reflective ways in which the computational mode of thinking is used, and the ways in which they coordinate the computational mode of thinking with other modes.

Pre-service Secondary Mathematics Teachers' Statistical Preparation: Interpreting the News

Joshua Chesler
Colorado D

Abstract: Undergraduate mathematics programs must prepare teachers for the challenges of teaching statistical thinking as advocated in standards documents and statistics education literature. This preliminary report presents initial results from a study of pre-service secondary mathematics teachers at the end of their undergraduate educations. Although nearly all had completed a required upper-division statistics course, most were challenged by two tasks which required a critical analysis of the use of statistics in newspaper articles. Some patterns emerged in the incorrect answers, including a tendency to focus on potential sampling issues which were not relevant to the tasks. The session will explore the nature and sources of these difficulties with statistical thinking and statistical communication and it will explore the implications for undergraduate mathematics and statistics teacher preparation.

The Effects of Formative Assessment on Students' Zone of Proximal Development in Introductory Calculus

Rebecca Dibbs and Michael Oehrtman

Gold Coin

Abstract: One of the challenges of teaching introductory calculus is the large variance in student backgrounds. Formative assessment can be used to target which students need help, but little is known about why formative assessment is effective with adult learners. The purpose of this qualitative study was to investigate which functions of formative assessment help instructors to provide the scaffolding needed to help students in an introductory calculus course progress through their Zones of Proximal Development during the weekly group labs. By providing students a low-stakes opportunity to demonstrate their current understanding, students were able to evaluate their progress and ask further questions after the activity was completed; this information was used to plan the discussion in the next class period. This discussion provided the scaffolding students needed to progress through the activities as well as providing peripheral participation opportunities for students who would not ordinarily ask questions during class.

Verifying Trigonometric Identities: Proof and Students' Perceptions of Equality

Benjamin Wescoatt

Matchless

Abstract: This preliminary study explores how students' perceptions of the equality of trigonometric expressions evolve during the process of verifying trigonometric identities (VTI). If students already view the purported equality as being true, VTI may not offer much in the way of learning experiences for students. Using a semiotic perspective to analyze student work, this study attempts to describe the evolution of students' perceptions of identities upon application of VTI, focusing on the components of the student's VTI process that contribute to the evolution. Initial analyses of interviews conducted while students proved identities indicate that students are not fully convinced that the identities are initially true. However, successful VTI, signaled, for example, by the use of an idiosyncratic equality construction, endows equality on not only the purported identity but on ancillary equality statements generated as part of the VTI process.

Student Responses to Team Based Learning in Tertiary Mathematics Courses

Judy Paterson, Louise Sheryn, and Jamie Sneddon

Colorado C

Abstract: Starting in 2009 we have implemented a Team Based Learning (TBL) model of delivery in two mathematics courses and one mathematics education course involving a total of 295 students. Qualitative data from evaluations, observations and interviews is used to begin to answer four questions raised by the Seldens (2001) regarding teaching mathematics at tertiary levels. Our analysis indicates that students say that TBL creates an environment in which they are active, have productive arguments and discussions and benefit from immediate feedback. There is scant evidence of any group being disadvantaged by this model of delivery.

Session 13 – Contributed Reports
2:20 – 2:50 PM

Venn Diagrams as Visual Representations of Additive and Multiplicative Reasoning in Counting Problems

Aviva Halani
Matchless

Abstract: This case study explored how a student could use Venn diagrams to explain his reasoning while solving counting problems. An undergraduate with no formal experience with combinatorics participated in nine teaching sessions during which he was encouraged to explain his reasoning using visual representations. Open coding was used to identify the representations he used and the ways of thinking in which he engaged. Venn diagrams were introduced as part of an alternate solution written by a prior student. Following this introduction, the student in this study often chose to use Venn diagrams to explain his reasoning and stated that he was envisioning them. They were a powerful model for him as they helped him visualize the sets of elements he was counting and to recognize over counting. Though they were originally introduced to express additive reasoning, he also used them to represent his multiplicative reasoning.

In-service Secondary Teachers' Conceptualization of Complex Numbers

Stephenie Anderson Dyben, Hortensia Soto-Johnson and Gulden Karakok
Denver 5 & 6

Abstract: This study explores in-service high school mathematics teachers' conception of various forms of a complex number and the ways that they transition between different representations' (algebraic and geometric) of these forms. Data were collected from three high school mathematics teachers via a ninety- minute interview after they completed professional development on complex numbers. Results indicate that these teachers do not necessarily objectify exponential form of complex numbers and only conceptualized it at the operational level. On the other hand, two teachers were very comfortable with Cartesian form and showed process/object duality by translating between different representations of this form. It appeared that our participants' ability to develop a dual conception of complex numbers was bound by their conceptualization of the various forms, which in turn was hindered by their representations of each form.

The Merits of Collaboration Between Mathematicians and Mathematics Educators on the Design and Implementation of An Undergraduate Course on Mathematical Proof and Proving

Orit Zaslavsky, Pooneh Sabouri, and Michael Thoms
Denver Ballroom 1-4

Abstract: The goal of our study was to characterize the processes and to identify the ways in which different kinds of expertise (mathematics vs. mathematics education) unfolded in the planning and teaching of an undergraduate course on Mathematical Proof and Proving (MPP), which was co-taught by a professor of mathematics and a professor of mathematics education. The content of the course consisted of topics that were supposed to be familiar to the students, i.e., high school level algebra, geometry, and basic number theory. In particular, we looked for

instances that would help understand how each expertise contributed to the course and complemented the other. The findings indicate that by co-teaching and constantly reflecting on their thinking and teaching, the instructors became aware of the added value of working together and the unique contribution each one had.

Teaching Undergraduate Calculus for Transfer: A Qualitative Case Study of the Calculus Sequence at One Liberal Arts College

Noelle Conforti Preszler

Gold Coin

Abstract: At small liberal arts colleges, a single calculus sequence must successfully accommodate students from various majors, such as mathematics, biology, chemistry, and economics. This qualitative case study considers mathematics professors' perspectives about the required nature of calculus in various disciplines, attempts to identify how calculus instructors teach with the aim of preparing students to apply calculus knowledge in their future coursework, and how the disciplinary focus of their students affects professors' design and teaching of calculus courses. Framed using aspects of teaching and learning shown to promote transfer of knowledge, results suggest that the professors teach for understanding and allow in-class processing time, but could improve their emphasis on applying calculus in non-mathematics disciplines. This study contributes to the growing body of undergraduate mathematics education research intended to document undergraduate teaching practices.

COFFE BREAK
2:50 – 3:20 PM
Denver Ballroom Pre-Function Area

Session 14 – Preliminary Reports
3:30 – 4:00 PM

Paradoxes of Infinity – The Case of Ken

Chanakya Wijeratne

Matchless

Abstract: Previous studies have shown that the normative solutions of the Pin-Pong Ball Conundrum and the Pin-Pong Ball Variation are difficult to understand even for learners with advanced mathematical background such as doctoral students in mathematics. This study examines whether this difficulty is due to the way they are set in everyday life experiences. Some variations of the Pin-Pong Ball Conundrum and the Pin-Pong Ball Variation and their abstract versions set in the set theoretic language without any reference to everyday life experiences were given to a doctoral student in mathematics. Data collected suggest that the abstract versions can help learners see beyond the metaphorical language of the paradoxes. The main contribution of this study is revealing the possible negative effect of the metaphorical language of the paradoxes of infinity on the understanding of the learner.

Self-inquiry in the Context of Undergraduate Problem Solving

Todd Grundmeier, Dylan Retsek and Dara Stepanek

Gold Coin

Abstract: Self-inquiry is the process of posing questions to oneself while solving a problem. The self-inquiry of thirteen undergraduate mathematics students was explored via structured interviews requiring the solution of both mathematical and non-mathematical problems. The students were asked to verbalize any thought or question that arose while they attempted to solve a mathematical problem and its nonmathematical logical equivalent. The thirteen students were volunteers who had each taken at least four upper division proof-based mathematics courses. Using transcripts of the interviews, a coding scheme for questions posed was developed and all questions were coded. While data analysis of the posed questions is ongoing, initial analysis suggests that the "good" mathematics students focus more questions on legitimizing their work and fewer questions on specification of the problem-solving task. Additionally, the self-inquiry of "fast" problem solvers mirrored that of the strong students with even less focus on specification questions.

Systematic Intuitive Errors on a Prove-or-Disprove Monotonicity Task

Kelly Bubp
Colorado C

Abstract: Despite the importance of intuitive and analytical reasoning in proof tasks, students have various difficulties with both types of reasoning. Such difficulties may be attributed to insufficient intuition, logical reasoning skills, or concept images. However, dual-process theory asserts that intuition can form faulty representations of tasks based on systematic errors before analytical reasoning can respond. Thus, students' difficulties could be attributed to systematic intuitive errors rather than inadequate intuitive or analytical reasoning. In this study, I conducted task-based interviews with four undergraduate and one graduate mathematics major in which they completed prove-or-disprove tasks. In this paper, I discuss the systematic intuitive errors committed by these students on a monotonicity task. These errors led all five students to believe incorrectly that the statement in the task was true. Furthermore, each student engaged in correct mathematical reasoning guided by their incorrect intuitive representations.

A Coding Scheme for Analyzing Graphical Reasoning on Second Semester Calculus Tasks

Rebecca Schmitz
Colorado D

Abstract: As a first step in studying students' spatial reasoning ability, preference, and their impact on performance in second semester calculus, I ran a pilot study to develop interview tasks and a coding scheme for analyzing the interviews. Four videotaped interviews were conducted with each of the five participants and the video was coded for graphical reasoning. I will discuss my coding scheme and share some preliminary results. I hypothesize that the coding scheme may help identify a student's preference and ability for spatial reasoning.

Mathematician's Tool Use in Proof Construction

Melissa Goss, Jeffrey King, and Michael Oehrtman
Denver 5 & 6

Abstract: The goals on teaching proof are to "help students develop an understanding of proof that is consistent with that shared and practiced by mathematicians of today." This study sought

to describe the tools and reasoning techniques used by mathematicians to construct and write proofs. Task-based clinical interviews were conducted with 3 research mathematicians in varying research fields. The tasks were upper-undergraduate and lower-graduate level proofs from linear algebra, basic analysis, and abstract algebra. Data were coded based on a framework constructed from Dewey's theory of inquiry and the characterizations of conceptual insight and technical handle. Preliminary results indicate the task of discovering a conceptual insight that can potentially lead to a proof can be problematic, and there are distinct moments in the construction process when the problem changes from "why should this be true?" to "how can I prove that?"

Initial Undergraduate Student Understanding of Statistical Symbols
Samuel Cook and Tim Fukawa-Connelly
Denver Ballroom 1-4

Abstract: In this study we use the tradition of semiotics to motivate an exploration of the knowledge of, and facility with, the symbol system of statistics that students bring to university. We collected a sample of incoming mathematics majors in their first semester of study, prior to taking any statistics coursework and engaged each in a task-based interview using a think-aloud protocol with questions designed to assess their fluency with basic concepts and symbols of statistics. Our findings include that students find symbols arbitrary and difficult to associate with the concepts. Second, that generally, no matter the amount of statistics that students took in high school, including Advanced Placement courses, they generally have relatively little recall of topics. Most can calculate the mean, median and mode, but they generally remember little beyond that. Finally, students have difficulty connecting practices or procedures to meaning.

Session 15 – Preliminary Reports
4:10 – 4:40 PM

Proof Structure in the Context of Inquiry Based Learning
Alyssa Eubank, Shawn Garrity and Todd Grundmeier
Denver Ballroom 1-4

Abstract: Data was collected from the final exams of 68 students in three sections of an introductory proofs course taught from an inquiry-based perspective. Inquiry-based learning (IBL) gives authority to students and allows them to present to their peers, rather than the instructor being the focus of the class and the authority on proof. This data was analyzed with a focus on proof structure. The selected final exam problems included concepts that were introduced prior to the course and others that were new to students. This research utilizes an adaptation of Toulmin's method for argumentation analysis. Our goal was to compare the proof structures generated by these students to previous research that also applied some form of Toulmin's scheme to mathematical proof. There was significant variety of proof structures, which could be a result of the IBL atmosphere.

Analyzing Calculus Concept Inventory Gains in Introductory Calculus
Matthew Thomas and Guadalupe Lozano
Gold Coin

Abstract: Research in science education, particularly physics education, indicates that students in Interactively-Engaged classrooms are more successful on tests of basic conceptual knowledge. Despite this, undergraduate mathematics courses are dominated by lectures in which students take a passive role. Given the value of such tests in assessing students' conceptual knowledge, the method for measuring such change is largely unexplored. In our study, students were given one such inventory, the Calculus Concept Inventory, in introductory Calculus classes as a pretest and posttest. We address issues of how gains might be measured on this instrument using two techniques, and the implications of using each of these measures.

A Modern Look at the Cell Problem

Jennifer Czocher
Colorado C

Abstract: Mathematical modeling perspectives continue to become viable lenses for examining students' mathematical thinking in novel contexts. Thus, it is vital to re-visit past foundational work in problem solving in order to connect new ideas and interpretations with accepted knowledge. The objective of this paper is to examine results from a well-known problem setting (The Cell Problem) to explore alternative interpretations of students' mathematical work.

Bringing the Familiar to the Unfamiliar: The Use of Knowledge from Different Domains in the Proving Process

Kathleen Melhuish
Matchless

Abstract: This report considers student proof construction in small groups within an inquiry-orientated abstract algebra classroom. During an initial analysis, several cases emerged where students used familiar knowledge from another mathematical domain to provide informal intuition. I will report on two episodes in order to illustrate how this intuition could potentially aid or hinder the construction of a valid proof.

Investigating Student Understanding of Eigentheory in Quantum Mechanics

Warren Christensen
Denver 5 & 6

Abstract: An initial investigation into students' understanding of Eigen theory using semi-structured interviews was conducted with students at the end of a first-semester course in quantum mechanics. Many physics faculty would expect students to have mastery of basic matrix multiplication after a course in Linear Algebra, and especially so after fairly extensive use of matrices in quantum mechanics in the context of Ising model spin problems. Using a previously published interview protocol by Henderson et al, student reasoning patterns were investigated to probe to what extent there reasoning patterns were similar to those identified among Linear Algebra students. Reasoning patterns appeared quite consistent with previous work; that is, students used superficial algebraic cancellation, and demonstrated difficulty interpreting their result even when they arrived at a correct solution. The interview protocol was

modified slightly to probe whether or not students felt the tasks they were engaging in were mathematical or physics-related. Additional questions were added at the end of the protocol about how these concepts were used in their quantum mechanics course. Students were somewhat successful relating them to Hamiltonians and energy eigenvalues, but couldn't articulate the type of physical situations where they might be useful.

Student Difficulties Setting Up Statistics Simulations in Tinkerplots™

Erin Glover and Jennifer Noll
Colorado D

Abstract: This preliminary report addresses the need for research that explores how technology changes the way students think about statistics and the ways technology can be used to enable students to construct models to solve statistical problems. This study focuses on student challenges interpreting a single trial of a statistical experiment and setting up TinkerplotsTM simulations. Sixteen students in a lower division introductory statistics course worked on a task involving the "One Son Policy", a situation in which families continue to have children until they have a boy. Students' interpretations of what a single trial represented in the One Son activity fell into four categories, three of which were completing the task. Further, students had difficulty with using the technology to set up and interpret a simulation to address the question. These results suggest that the process of setting up a computer simulation to answer a statistical question is quite complex.

Session 16 – Contributed Reports
4:50 – 5:20 PM

Students' Axiomatizing in a Classroom Setting

Mark Yannotta
Colorado C

Abstract: The purpose of this paper is to examine descriptive axiomatizing as a classroom mathematical activity. More specifically, if given the opportunity, how do students select axioms and how might their intellectual needs influence these decisions? These two case studies of axiomatizing address these questions and elaborate on how students engage in this practice within a classroom setting. The results of this research suggest that while students may at first be resistant to axiomatizing, this mathematical activity also affords them opportunities to create meaning for new mathematical content and for the axiomatic method itself.

Understanding Abstract Algebra Concepts

Anna Titova
Matchless

Abstract: This study discusses various theoretical perspectives on abstract concept formation. Students' reasoning about abstract objects is described based on proposition that abstraction is a shift from abstract to concrete. Existing literature suggested a theoretical framework for the study. The framework describes process of abstraction through its elements: assembling, theoretical generalization into abstract entity, and articulation. The elements of the theoretical

framework are identified from students' interpretations of and manipulations with elementary abstract algebra concepts including the concepts of binary operation, identity and inverse element, group, subgroup. To accomplish this, students participating in the abstract algebra class were observed during one semester. Analysis of interviews and written artifacts revealed different aspects of students' reasoning about abstract objects. Discussion of the analysis allowed formulating characteristics of processes of abstraction and generalization. The study offers theoretical assumptions on students reasoning about abstract objects. The assumptions, therefore, provide implications for instructions and future research.

Covariational Reasoning and Graphing in Polar Coordinates
Kevin Moore, Teo Paoletti, Jackie Gammaro, and Stacy Musgrave
Gold Coin

Abstract: An extensive body of research exists on students' function concept in the context of graphing in the Cartesian coordinate system (CCS). In contrast, research on student thinking in the context of the polar coordinate system (PCS) is sparse. In this report, we discuss the findings of a teaching experiment that sought to characterize two undergraduate students' thinking when graphing in the PCS. As the study progressed, the students' capacity to engage in covariational reasoning emerged as critical for their ability to graph relationships in the PCS. Additionally, such reasoning enabled the students to understand graphs in the CCS and PCS as representative of the same relationship despite differences in appearance. Collectively, our findings illustrate the importance of covariational reasoning for conceiving graphs as relationships between quantities' values and that graphing in the PCS might create one opportunity to promote such reasoning when combined with graphing in the CCS.

Using Metaphors to Support Students' Ability to Reason about Logic
Paul Christian Dawkins and Kyeong Hah Roh
Denver 5 & 6

Abstract: In this paper, we describe an inquiry-oriented method of using metaphors to support students' development of conventional logical reasoning in advanced mathematics. Our model of instruction was developed to describe commonalities observed in the practice of two inquiry-oriented real analysis instructors. We present the model via a general thought experiment and one representative case study of a students' metaphorical reasoning. Part of the success of the instructional method relates to its ability to help students reason about, assess, and communicate about the logical structure of mathematical activity. In the case presented, this entailed a students' shift from using properties to describe examples to using examples to relate various properties. The metaphor thus imbued key example sequences with meta-theoretical significance. We introduce the term "wedge" to describe such examples that distinguish oft-conflated properties. We also present our analytical criteria for empirically verifying the specific influence of the metaphorical aspect of instruction.

PLENARY SESSION

5:30 – 6:30 PM
Denver Ballroom 1-4

How to Support Students in Constructing More Formal Mathematics
Dr. Koeno Gravemeijer
Eindhoven University of Technology

Abstract: Formal mathematical knowledge is hard to transmit to students. For it typically concerns knowledge on a higher level of understanding than the students possess. Instead of trying to transmit mathematical knowledge, we may aim at helping students to construct new mathematical knowledge by building on what they already know. This presents mathematics educators with the difficult task of building on the informal knowledge of the students while working towards the conventional formal mathematics one is aiming for. I will discuss the theory of realistic mathematics education (RME) that offers guidelines on how to reconcile informal, experientially real starting points with conventional, abstract, mathematical endpoints. Key here are processes of generalizing and formalizing. There are, however, various pitfalls that threaten the realization of such processes in regular classrooms.

DINNER ON YOUR OWN

SATURDAY
February 23, 2013

Session 17 – Preliminary Reports
8:30 – 9:00 AM

Providing Opportunities for College-Level Calculus Students to Engage in Theoretical Thinking
Dalia Challita and Nadia Hardy
Gold Coin

Abstract: Previous research has reported an absence of a theoretical thinking component in college-level Calculus courses; moreover, valid arguments can be made for or against the necessity and feasibility of incorporating such a component. Our belief is, however, that students who wish to engage in theoretical thinking should be given the chance to do so in such a course. The current report presents a preliminary analysis of a study we conducted in a Calculus class in which we presented students with tasks, in the form of quizzes, intended to provoke a type of behavior that is indicative of theoretical thinking. Using Sierpinska et al.'s (2002) model as a basis for theoretical thinking we show that students were indeed engaged in theoretical thinking through these tasks. Our preliminary analysis of the results suggests that despite constraints often faced by instructors of such courses, incorporating such a component is indeed feasible.

Students' Knowledge Resources About the Temporal Order of Delta and Epsilon
Aditya Adiredja and Kendrice James
Colorado C

Abstract: The formal definition of a limit, or the epsilon delta definition is a critical topic in

calculus for mathematics majors' development and the first chance for students to engage with formal mathematics. Research has documented that the formal definition is a roadblock for most students but has de-emphasized the productive role of their prior knowledge and sense making processes. This study investigates the range of knowledge resources included in calculus students' prior knowledge about the relationship between delta and epsilon within the definition. diSessa's Knowledge in Pieces provides a framework to explore in detail the structure of students' prior knowledge and their role in learning the topic.

Two Students' Interpretation of Rate of Change in Space

Eric Weber

Denver Ballroom 1-4

Abstract: This paper describes a model of the understandings of two first-semester calculus students, Brian and Neil, as they participated in a teaching experiment focused on exploring ways of thinking about rate of change of two-variable functions. I describe the students' construction of directional derivative as they attempted to generalize their understanding of one-variable rate of change functions, and characterize the importance of quantitative and covariational reasoning in this generalization.

Conceptualizing Vectors in College Geometry: A New Framework for Analysis of Student Approaches and Difficulties

Oh Hoon Kwon

Matchless

Abstract: This article documents a new way of conceptualizing vectors in college geometry. The complexity and subtlety of the construct of vectors highlight the need for a new framework that permits a layered view of the construct of vectors. The framework comprises three layers of progressive refinements: a layer that describes a global distinction between physical vectors and mathematical vectors, a layer that recounts the difference between the representational perspective and the cognitive perspective, and a layer that identifies ontological and epistemological obstacles in terms of transitions towards abstraction. Data was gathered from four empirical studies with ninety-eight total students to find evidence of the three major transition points in the new framework: physical to mathematical coming from the first layer, geometric to symbolic and analytic to synthetic from the second layer, and the prevalence of the analytic approach over the synthetic approach while developing abstraction enlightened by the third layer.

Jump Math Approach to Teaching Foundations Mathematics in 2-Year College Shows Consistent Gains in Randomized Field Trial

Taras Gula and Carolyn Hoessler

Colorado D

Abstract: Many first year college students struggle with foundational mathematics skills even after one semester of mathematics. JUMP math, a systematized program of teaching mathematics, claims that its approach, though initially designed for K-8, can strengthen skills at the foundations college math level as well. Students in sixteen sections of Foundations

Mathematics at a college in Canada were randomly assigned to be taught with either the JUMP math approach or a typical teaching approach. Students were measure before and after on their competence (Wechsler test of Numerical Operations) and attitudes (Mathematics Attitudes Inventory) to identify any improvements. Results showed that students in JUMP classes had modest, but consistently higher improvements in competence when compared to students in non-JUMP classes, even after controlling for potential confounding variables, while improvements in Math Attitudes showed no differences.

Identifying Change in Secondary Mathematics Teachers' Pedagogical Content Knowledge
Melissa Goss, Robert Powers, and Shandy Hauk
Denver 5 & 6

Abstract: Like several other research groups, we have been investigating multiple measures for capturing change in middle and high school teachers' mathematical pedagogical content knowledge (PCK). This article reports on results among 14 teachers (of 16 enrolled) who have completed a virtual master's program in mathematics education. The degree program seeks to develop content proficiency, cultural competence, and pedagogical expertise for teaching mathematics. Analysis included pre- and post-program data from classroom observations and written PCK assessment. Results indicate significant changes in curricular content knowledge on the observation instrument and significant changes in discourse knowledge on both the observation instrument and the written assessment. Additional path analysis suggests teacher discourse knowledge as measured by the written assessments is significantly related to discourse knowledge as measured by the post-program observation.

Session 18 – Contributed Reports
9:10 – 9:40 AM

A Dialogic Method of Presenting Proofs: Focus on Fermat's Little Theorem
Boris Koichu and Rina Zazkis
Denver 5 & 6

Abstract: Twelve participants were asked to decode a proof of Fermat's Little Theorem and present it in a form of a script for a dialogue between two characters of their choice. Our analysis of these scripts focuses on issues that the participants identified as 'problematic' in the proof and on how these issues were addressed. Affordances and limitations of this dialogic method of presenting proofs are exposed, by means of analyzing how the students' correct, partial or incorrect understanding of the elements of the proof are reflected in the dialogues. The difficulties identified by the participants are discussed in relation to past research on undergraduate students' difficulties in proving and in understanding number theory concepts.

On the Role of Pedagogical Content Knowledge in Teachers' Understanding of Commutativity and Associativity
Steven Boyce
Matchless

Abstract: The purpose of this study is to investigate a relationship between mathematical content knowledge and pedagogical knowledge of content and students (Hill, Ball, & Shilling,

2008), in the context of algebra. As participants in a paired teaching experiment, mathematics education doctoral students revealed their understandings of commutativity and associativity (cf. Larsen, 2010). Although the participants' knowledge of children's initial understandings of algebra and familiarity with mathematics education literature influenced their own mathematics reasoning, the difficulties they encountered were similar to those of undergraduates without such pedagogical content knowledge.

Developing Facility with Sets of Outcomes by Solving Smaller, Simpler Counting Problems
Elise Lockwood
Denver Ballroom 1-4

Abstract: Combinatorial enumeration has a variety of important applications, but there is much evidence indicating that students struggle with solving counting problems. In this paper, the use of the problem-solving heuristic of solving smaller, similar problems is tied to students' facility with sets of outcomes. Drawing upon student data from clinical interviews in which post-secondary students solved counting problems, evidence is given for how numerical reduction of parameters can allow for a more concrete grasp of outcomes. The case is made that the strategy is particularly useful within the area of combinatorics, and avenues for further research are discussed.

Pre-Service Secondary Teachers' Meanings for Fractions and Division
Cameron Byerley and Neil Hatfield
Gold Coin

Abstract: In this study, seventeen math education majors completed a test on fractions and quotient. From this group, one above-average calculus student was selected to participate in a six-lesson teaching experiment. The major question investigated was "what constrains and affords the development of the productive meanings for division and fractions articulated by Thompson and Saldanha (2003)?" The student's thinking was described using Steffe and Olive's (2010) models of fractional knowledge. The report focuses on the student's part-whole meaning for fractions and her difficulty assimilating instruction on partitive meanings for quotient. Her part-whole meaning for fractions led to the resilient belief that any partition of a length of size m must result in m, unit size pieces. It was non-trivial to develop the basic meanings underlying the concept of rate of change, even with a future math teacher who passed calculus.

COFFEE BREAK
9:50 – 10:20 AM
Denver Ballroom Pre-Function Area

Session 19 – Preliminary Reports
10:30 – 11:00 AM

Difficulties in Using Variables – A Tertiary Transition Study
Ileana Borja-Tecuatl, Asuman Oktaç and María Trigueros
Colorado C

Abstract: This article describes the results obtained from a diagnostic instrument to establish the difficulties in understanding and using variables that engineering student's have at the moment

of their entrance to a public Mexican university, that does not examine the candidates prior to admittance. Once the difficulties were established, a 1-year treatment based on the 3 Uses of Variables Model was applied to foster a rich conception of variable in the students. The purpose of this treatment was on one hand to enrich the students' concept of variable, to make it possible that they consider variables as dynamic objects that not only represent unknowns, but also describe relations between the objects they represent, and that may vary their usage along one same problem. On the other hand, we wanted to set the basis to study how a poor/rich conception of variable interferes with understanding the solution of a linear equations system.

Gestures: A Window to Mental Model Creation
Nancy Garcia and Nicole Engelke
Denver 5 & 6

Abstract: Gestures are profoundly integrated into communication. This study focuses on the impact that gestures have in a mathematical setting, specifically in an undergraduate calculus workshop. We identify two types of gesture – dynamic and static – and note a strong correlation between these movements and diagrams produced. Gesture is a primary means for students to communicate their ideas to each other, giving them a quick way to share thoughts of relative motion, relationships, size, shape, and other characteristics of the problem. Dynamic and static gestures are part of the students' thinking, affecting how they view the problem, sway group thinking, and the construction of their diagrams.

Development of Students' Ways of Thinking in Vector Calculus
Eric Weber and Allison Dorko
Gold Coin

Abstract: In this talk, we describe the development of the ways of thinking of 25 vector calculus students over the course of one term. In particular, we characterize the generalizations that students made within and across interviews. We focus on the construction of the semi-structured pre and post interviews, trace the construction of explanatory constructs about student thinking that emerged from those interviews, and describe how those constructs fit within the broader literature on student thinking in advanced calculus. We conclude by exploring implications for future research and practical applications for educators.

Characteristics of Successful Programs in College Calculus: Pilot Case Study
Sean Larsen, Estrella Johnson and Steve Strand
Denver Ballroom 1-4

Abstract: The CSPCC (Characteristics of Successful Programs in College Calculus) project is a large empirical study investigating mainstream Calculus 1 to identify the factors that contribute to success, to understand how these factors are leveraged within highly successful programs. Phase 1 of CSPCC entailed large-scale surveys of a stratified random sample of college Calculus 1 classes across the United States. Phase 2 involves explanatory case study research into programs that are successful in leveraging the factors identified in Phase 1. Here we report preliminary findings from a pilot case study that was conducted at a private liberal arts university. We briefly describe the battery of interviews conducted at the pilot site and discuss

some of the themes that have emerged from our initial analyses of the interview data.

Cooperative Learning and Traversing the Continuum of Proof Expertise
Martha Byrne
Colorado D

Abstract: This paper describes preliminary results of a study aimed at examining the effects of working in cooperative groups on acquisition and development of proof skills. Particular attention will be paid to the varying tendencies of students to switch proof methods (direct, induction, contradiction, etc) based on their level of proof expertise.

Students' Sense-Making in Mathematics Lectures
Aaron Weinberg, Tim Fukawa-Connelly, and Emilie Wiesner
Matchless

Abstract: Many mathematics students experience proof-based classes primarily through lectures, although there is little research describing what students actually learn from such classroom experiences. Here we outline a framework, drawing on the idea of the implied observer, to describe lecture content; and apply the framework to a portion of a lecture in an abstract algebra class. Student notes and interviews are used to investigate the implications of this description on students' opportunities to learn from proof-based lectures. Our preliminary findings detail the behaviors, codes, and competencies that an algebra lecture requires. We then compare those with how students behave in response to the same lecture with respect to sense-making and note-taking, and thereby how they approach opportunities to learn.

Session 20 – Contributed Reports
11:10 – 11:40 AM

Preservice Teachers' Mathematical Knowledge for Teaching and Concepts of Teaching Effectiveness: Are They Related?
Jathan Austin
Colorado D

Abstract: Mathematical knowledge for teaching (MKT) is essential for effective teaching of elementary mathematics. Given the importance of MKT, MKT and conceptions of teaching effectiveness should not develop independently. The purpose of this study was to examine whether and how K-8 pre-service teachers' MKT and personal mathematics teacher efficacy beliefs are related. Results indicated overconfidence in teaching ability was prevalent, with the majority of participants exhibiting a strong sense of personal mathematics teacher efficacy but low levels of MKT. Pre-service teachers with high levels of MKT, however, reported a more accurate assessment of their teaching effectiveness. Results also indicated that examining pre-service teachers' self-evaluations of MKT is helpful for understanding pre-service teachers' personal mathematics teacher efficacy beliefs. Moreover, the results of this study point to the inadequacies of existing measures of teacher efficacy beliefs that do not parse out differences in efficacy beliefs according to a number of contextual factors.

Students' Emerging Understandings of the Polar Coordinate System

Teo Paoletti, Kevin Moore, Jackie Gammaro, and Stacy Musgrave
Gold Coin

Abstract: The Polar Coordinate System (PCS) arises in a multitude of contexts in undergraduate mathematics. Yet, there is a limited body of research investigating students' understandings of the PCS. In this report, we discuss findings from a teaching experiment concerned with exploring four pre-service teachers' developing understandings of the PCS. We illustrate the role students' meanings for angle measure played while constructing the PCS. Specifically, students with a stronger understanding of radian angle measure more fluently constructed the PCS than their counterparts. Also, we found that various aspects of the students' understandings of the Cartesian coordinate system (CCS) became problematic as they transitioned to the PCS. For instance, mathematical differences between the polar pole and Cartesian origin presented the students difficulties. Collectively, our findings highlight important understandings that can support or prevent students from developing a robust conception of the PCS.

Understanding Mathematical Conjecturing

Jason Belnap and Amy Parrott
Denver 5 & 6

Abstract: In this study, we open up discussions regarding one of the unexplored aspects of mathematical sophistication, the inductive work of conjecturing. We consider the following questions: What does conjecturing entail? How do the conjectures of experts and novices differ? What characteristics, behaviors, practices, and viewpoints distinguish novice from expert conjecturers? and What activities enable individuals to make conjectures? To answer these questions, we conducted a qualitative research study of eight participants at various levels of mathematical maturity. Answers to our research questions will begin to provide an understanding about what helps students develop the ability to make mathematical conjectures and what characteristics of tasks and topics may effectively elicit such behaviors, informing curriculum development, assessment, and instruction.

Building Knowledge for Teaching Rates of Change:
Three Cases of Physics Graduate Students

Natasha Speer and Brian Frank
Matchless

Abstract: Over the past two decades education researchers have demonstrated that various types of knowledge, including pedagogical content knowledge, influence teachers' instructional practices and their students' learning opportunities. Findings suggest that by engaging in the work of teaching, teachers acquire knowledge of how students think, but we have not yet captured this learning as it occurs. We examined whether novice instructors can develop such knowledge via the activities of attending to student work and we identified mechanisms by which such knowledge development occurs. Data come from interviews with physics graduate teaching assistants as they examined and discussed students' written work on problems involving rates of change. During those discussions, some instructors appear to develop new knowledge–either about students' thinking or about the content—and others did not. We compare and contrast three cases representing a range of outcomes and identify factors that enabled some instructors to build new knowledge.

LUNCH
11:50 – 12:50 AM
Denver Ballroom 1-4

PLENARY SESSION
1:00 – 2:00 PM
Denver Ballroom 1-4

How Visualization Techniques Shape the Landscape of Mathematics and Science
Dr. Loretta Jones
University of Northern Colorado

Abstract: Dynamic visualizations of scientific and mathematical processes can be powerful learning tools. However, their use has not always enhanced learning and, in fact, has sometimes been found to mislead learners. Collaborations among the research communities of cognitive science, education, mathematics, and physical and life sciences are helping science and mathematics education researchers to study the instructional uses of visualization techniques. These collaborations reveal how students perceive and interpret various kinds of visualizations and animations and show how to develop design principles for creating and using effective instructional visualizations. This presentation will examine research paradigms developed for investigating these areas.

COFFEE BREAK
2:10 – 2:40 PM
Denver Ballroom Pre-Function Area

Session 21 – Theoretical Reports
2:50 – 3:20 PM

Does/Should Theory Building Have a Place in the Mathematics Curriculum?
Hyman Bass
Denver Ballroom 1-4

Abstract: Mathematicians distinguish two modes of their practice – problem solving and theory building. While problem solving has a robust presence in the mathematics curriculum, it is less clear whether theory building does, or should, have such a place. I will report on a curricular design to support a kind of simulation of mathematical theory building. It is based on the notion of a "common structure problem set" (CSPS). This is a small set of mathematical problems with a two-part assignment: I. Solve the problems; and II. Find and articulate a mathematical structure common to all of them. Some examples will be presented and analyzed. Relations of this construct to earlier ideas in the literature will be presented, in particular to the notion of isomorphic problems, and cognitive transfer. Designing effective instructional enactments of a CSPS is still very much in an experimental stage, and feedback about this would be welcome.

Developing an Explication Analytical Lens for Proof-oriented Mathematical Activity
Paul Christian Dawkins
Denver 5 & 6

Abstract: Sjogren (2010) suggested that formal proof could be understood as an explication (Carnap, 1950) of informal proof. Explication describes the supplanting of an intuitive or

unscientific concept by a scientific or formal concept. I clarify and extend Sjogren's claim by applying Carnap's criteria for explication (similarity, exactness, and fruitfulness) to definitions, theorems, axioms, and proofs. I synthesize a range of proof-oriented research constructs into one overarching framework for representing and analyzing students' proving activity. I also explain how the analytical framework is useful for understanding student difficulties by outlining some results from an undergraduate, neutral axiomatic geometry course. I argue that mathematical contexts like geometry in which students have strong spatial and experiential intuitions may require successful semantic style reasoning. This demands that students' construct rich ties between different representation systems (verbal, symbolic, logical, imagistic) justifying explication as a reasonable analytical lens for this and similar proof-oriented courses.

The Action, Process, Object, and Schema Theory of Sampling
Neil Hatfield
Gold Coin

Abstract: This paper puts forth a new theoretical perspective for students' understanding of sampling. The Action, Process, Object, and Schema Theory for Sampling serves as a potential bridge between Saldanha's and Thompson's Multiplicative Conception of Sampling and APOS Theory. This theoretical perspective provides one potential way to describe the development of a student's conception of sampling. Additionally this perspective differs from most other perspectives in that it does not focus on the sample size the student uses or the sampling method, but rather how the student understands sampling in terms of a sampling distribution.

Illustrating a Theory of Pedagogical Content Knowledge for Secondary and Post-Secondary Mathematics Instruction
Shandy Hauk, Allison Toney, Billy Jackson, Reshmi Nair, and Jengq-Jong Tsay
Matchless

Abstract: The accepted framing of pedagogical content knowledge (PCK) as mathematical knowledge for teaching has centered on the question: What mathematical reasoning, insight, understanding, and skills are required for a person to teach mathematics? Many have worked to address this question, particularly among K-8 teachers. What about teachers with broader mathematics knowledge (e.g., from algebra to proof-based understandings of topics in advanced mathematics)? There is a need for examples and theory in the context of teachers with greater mathematical preparation and older students with varied and complex experiences in learning mathematics. This theory development piece offers background and examples for an extended theory of PCK as the interplay among conceptually-rich mathematical understandings, experience of teaching, and multiple culturally-mediated classroom interactions.

Session 22 – Contributed Reports
3:30 – 4:00 PM

Undergraduate Students' Models of Curve Fitting
Shweta Gupta
Denver Ballroom 1-4(updated)

Abstract: The Models and Modeling Perspectives (MMP) has evolved out of research that

began 26 years ago. MMP research uses Model Eliciting Activities (MEAs) to elicit students' models of mathematical concepts. In this study MMP were used as conceptual framework to investigate the nature of undergraduate students' models of curve fitting. Participants of this study were prospective mathematics teachers enrolled in an undergraduate mathematics problem solving course. Videotapes of the MEA session, class observation notes, and anecdotes from class discussions served as the sources of data for this study. Iterative videotape analyses as described in Lesh and Lehrer (2003) were used to analyze the videotapes of the participants working on the MEA. Results of this study discuss the nature of students' models of the concept of curve fitting and add to the introductory undergraduate statistics education research by investigating the learning of the topic curve fitting.

On the Sensitivity of Problem Phrasing – Exploring the Reliance of Student Responses on Particular Representations of Infinite Series

Danielle Champney

Denver 5 & 6 (updated)

Abstract: This study will demonstrate the ways in which students' ideas about convergence of infinite series are deeply connected to the particular representation of the mathematical content, in ways that are often conflicting and self-contradictory. Specifically, this study explores the different limiting processes that students attend to when presented with five different phrasings of a particular mathematical task - ∑(1/2)^n - and the ways in which each phrasing of the task brings to light different ideas that were not evident or salient in the other phrasings of the same task. This research suggests that when attempting to gain a more robust understanding of the ways that students extend the ideas of calculus – in this case, limit – one must take care to attend to not only students' reasoning and explanation, but also the implications of the representations chosen to probe students' conceptions, as these representations may mask or alter student responses.

Session 23 – Preliminary Reports
4:10 – 4:40 PM

Guided Reflections on Mathematical Tasks: Fostering MKT in College Geometry

Josh Bargiband, Sarah Bell and Tetyana Berezovski

Gold Coin

Abstract: This study is a part of ongoing research on development of Mathematical Knowledge for Teaching (MKT) in mathematical content courses. Reflective practice represents a central theme in teacher education. The purpose of this reported study was to understand the role of guided reflections on mathematical tasks in a college geometry course. We were also interested in understanding how guided reflections on mathematical tasks would effect teachers' development of MKT. Our research data consist of participants' reflections, teaching scenarios, and pre-post test results. In this study we developed a workable framework for data analysis. Audience discussion will address questions related to the proposed analysis framework and development of MKT in college mathematics courses.

Effects of Collaborative Revision on Beliefs about Proof Function and Validation Skills

Emily Cilli-Turner

Colorado C

Abstract: Although there is much research showing that proof serves more than just a verification function in mathematics, there is little research documenting which functions of proof undergraduate students understand. Additionally, research suggests that students have difficulty in determining the validity of a given proof. This study examines the effects of a teaching intervention called collaborative revision on student beliefs regarding proof and on student proof validation skills. Student assessment data was collected and interviews were conducted with students in the treatment course and in a comparison course. At the end of this study, we will produce a categorization of the proof functions that students appreciate, as well as a determination of the value of the teaching intervention on students' abilities to correctly classify proofs as valid or invalid.

Assessing Student Presentations in an Inquiry-Based Learning Course

Christina Eubanks-Turner

Colorado D

Abstract: Inquiry-Based Learning (IBL) is an instruction method that puts the student as the focal point of the learning experience. An integral part of many IBL proof-based courses is presentation of proofs given by students. In this report, I introduce an assessment rubric used to evaluate student presentations of theoretical exercises from a presentation script. The W.I.P.E. rubric is based on several different assessment models, which emphasize proof writing and comprehension. The rubric has been created to evaluate in-class presentations in undergraduate Abstract Algebra courses for math and math education majors, which offer graduate credit.

The Role of Time in a Related Rate Scenario

Katherine Czeranko

Matchless

Abstract: Students who graduate with an engineering or science degree using applied mathematics are expected to synthesize concepts from calculus to solve problems. First semester calculus students attempting to understand the derivative as a rate of change encounter difficulties. Specifically, the challenges arise while making the decision to apply an average rate of change or an instantaneous rate of change (Zandieh, 2000) to the problem. This paper discusses how students view the derivative in an applied mathematical setting and investigates how the concept of time and other related quantities contribute to the development of a solution.

Calculus Students' Understanding of Volume

Allison Dorko and Natasha Speer

Denver 5 & 6

Abstract: Understanding the concept of volume is important to the learning/understanding of various topics in undergraduate mathematics. Researchers have documented difficulties that elementary school students have in understanding volume, but we know little about college students' understanding of this topic. This study investigated calculus students' volume understanding. Clinical interview transcripts and written responses to volume problems were

analyzed. Findings include: (1) some calculus students find surface area when directed to find volume, believing that the surface area computation accounts for the object's three-dimensional space; and (2) some calculus students find volume using reasoning and formulae that contain surface area and volume elements. Comparisons with research on elementary school students' thinking and implications for the teaching and learning of volume-dependent topics from calculus are also presented.

BREAK
5:00 – 6:00 PM

AWARDS BANQUET & PLENARY SESSION
6:00 – 9:00 PM
Denver Ballroom 1-4

PLENARY TALK:
Benefits and Limitations of Complimenting Qualitative Research with Quantitative Studies: Lessons Learned from Five Studies on Proof Reading
Dr. Keith Weber
Rutgers University

Abstract: Most mathematics educators agree that research in our discipline is most effective when it combines theoretical, qualitative, and quantitative approaches. Yet the research presented at the annual RUME conferences is dominated by qualitative studies, with research reports on quantitative studies being relatively rare. Until recently, this trend was present in my research projects as well, which nearly exclusively employed qualitative studies. Over the last few years, I have begun complimenting this qualitative work with quantitative studies. The purpose of this presentation is to describe what I have learned through this process in the context of five quantitative studies on the reading of mathematical proof. The findings represent my views of how quantitative studies can compliment qualitative research, including how such studies can be conducted and the benefits and limitations in engaging in this type of research. I conclude that quantitative research is a useful but underused methodology in the RUME community.